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05 Properties of Matter and Fluid Mechanics

Mechanical Properties of Solids

Introduction to types of forces and types of bodies

Types of forces

Deforming Forces

External force which tends to bring about a change in the length, volume or shape of a body is called deforming force.

Restoring Forces

When an external force acts at any object then an internal resistance produced in the material due to the intermolecular forces which is called restoring force.

Elasticity

Elasticity is that property of a material of a body by virtue of which it opposes any change in its shape or size when deforming forces are applied on it, and recover its original state as soon as the deforming force are removed

Types of Bodies

Rigid Body

A body is said to be rigid if the relative positions of its constituent particles remain unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Perfectly Elastic Body

A body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Example : quartz. – It is quite close to a perfect elastic body.

Plastic Body

A body which does not have the property of opposing the deforming forces, is known as a plastic body. All bodies which remain in the deformed state even after the removal of the deforming forces are known as plastic bodies. Example : clay, wax, putty.

Stress and its types, breaking stress

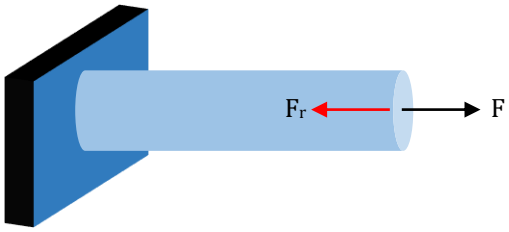
Stress

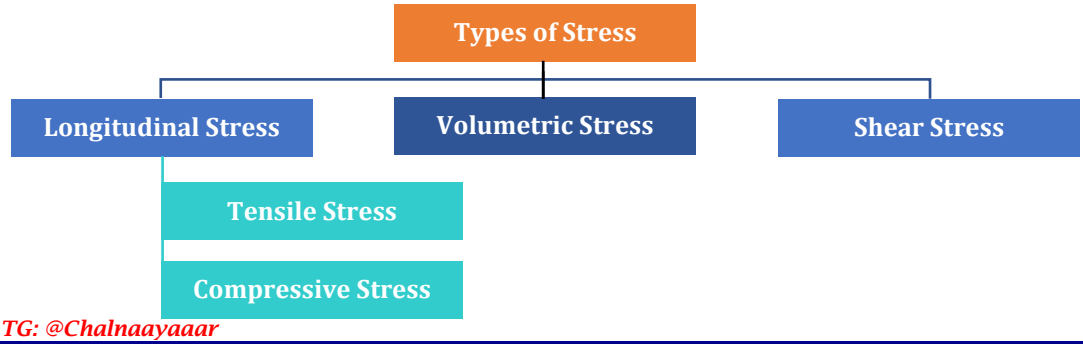
The restoring force acting per unit area of the deformed body is called **stress**.

$$\text{stress} = \frac{\text{internal restoring forces}}{\text{area of cross section}}$$

$$\text{stress} = \frac{F_{external}\,(\text{at equilibrium})}{A}$$

SI Unit: N/m^2 Dimensions: $[M^1L^{-1}T^{-2}]$





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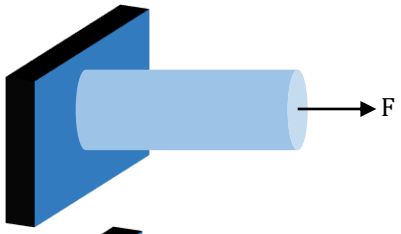
Types of stress

(i) Longitudinal Stress

When the stress is normal to the cross sectional area, then it is known as **longitudinal stress.**

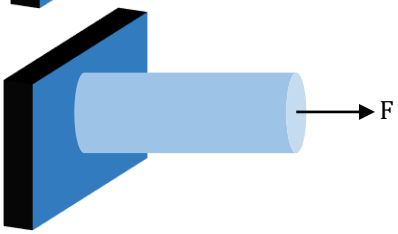


$$\text{Longitudinal Stress} = \frac{F_{\perp}}{A}$$

There are two types of longitudinal stress:

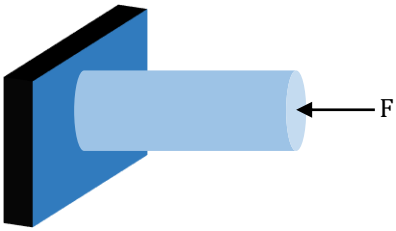
(a) Tensile Stress

The longitudinal stress, produced due to increase in the length of a body, is defined as tensile stress.



(b) Compressive Stress

The longitudinal stress, produced due to the decrease in the length of a body, is defined as compressive stress.



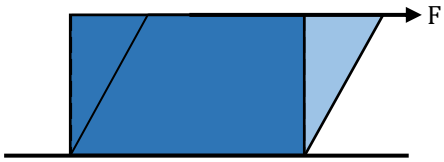
(ii) Volume Stress/Hydraulic Stress

If equal normal forces are applied over every unit surface of a body, then it undergoes a certain change in volume. The force opposing this change in volume per unit area is defined as volume stress.

$$\text{Volume Stess/Hydraulic Stress} = \frac{F_{\perp}}{A} = P$$

(iii) Shear Stress/Tangential Stress

When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted but not its volume.

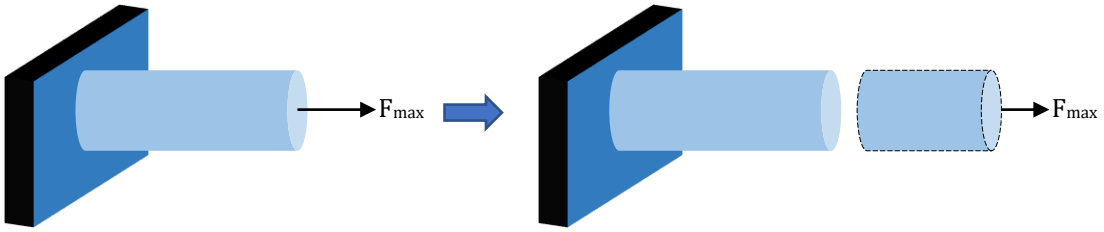


$$\text{Shear stress} = \frac{F_t}{A} = \frac{F_{\text{Tangential}}}{A}$$

(iv) Breaking Stress

The minimum stress required to cause the actual fracture of a material is called the **breaking stress** or ultimate strength. **or**

The maximum stress that a material can bear is called breaking stress.



$$\text{Breaking stress} = \frac{F_{max}}{A}$$ F_{max} = force required to break the body.

Dependence of breaking stress:

(i) Nature of material (ii) Temperature (iii) Impurities.

Independence of breaking stress :

(i) Cross sectional area or thickness (ii) Applied force (F_{max} = B.S. × Area).

Maximum load (force) which can applied on the wire depends on

(i) Cross sectional area or thickness (ii) Nature of material

(iii) Temperature (iv) Impurities

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Illustration 1:

The ratio of radii of two wires of same materials is 2 : 1. Find if they are stretched by the same force, the ratio of stress:

Solution:

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{\pi r^2} \Rightarrow \frac{(\text{stress})_1}{(\text{stress})_2} = \frac{F}{\pi r_1^2} \times \frac{\pi r_2^2}{F} = \left[\frac{r_2}{r_1}\right]^2 = \left[\frac{1}{2}\right]^2 = \frac{1}{4}$$

Illustration 2:

A body of mass 10 kg is attached to a 30 cm long wire whose breaking stress is 4.8×10^7 N/m^2. The area of cross section of the wire is 10^{-6} m^2. What is the maximum angular velocity with which it can be rotated in a horizontal circle?

Solution:

$$\frac{m\omega^2 L}{A} = \text{breaking stress (BS)} \Rightarrow \omega = \sqrt{\frac{(BS)A}{mL}} = \sqrt{\frac{4.8\times10^7\times10^{-6}}{10\times0.3}} = 4 \text{ rad/s}$$

Illustration 3:

The breaking stress of aluminium is 7.5×10^8 dyne/cm^2. Find the maximum length of aluminium wire that can hang vertically without getting broken under its own weight. Density of aluminium is 2.7 g/cm^3.
Given : g = 980 cm/s^2.

Solution:

Let L be the maximum length of the wire that can hang vertically without getting broken.
Mass of the wire, m = cross-sectional area (A) × length (L) × density (ρ)
Weight of the wire = mg = ALρg
This is equal to the maximum force that the wire can withstand.

$$\therefore \text{ Breaking stress} = \frac{LA\rho g}{A} = L\rho g$$

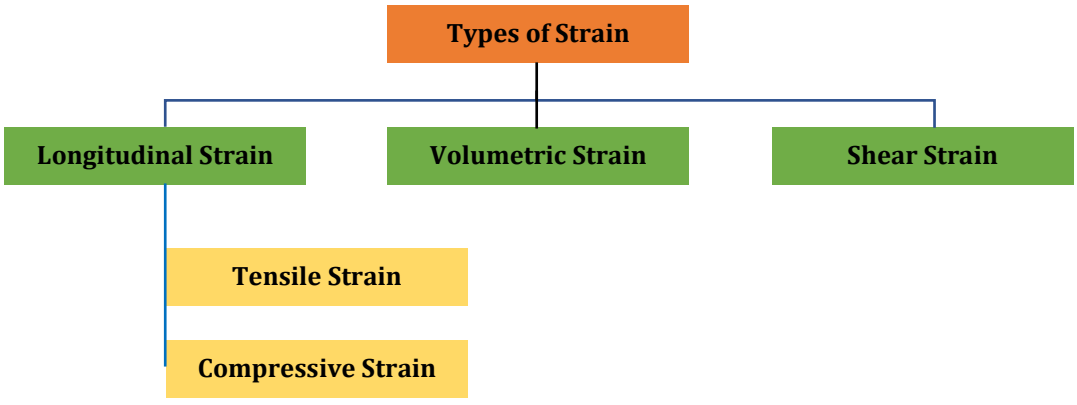
$$\text{Or } 7.5 \times 10^8 = L \times 2.7 \times 980 \Rightarrow L = \frac{7.5\times10^8}{2.7\times980} = 2.834 \times 10^5 \text{ cm} = 2.3 \text{ km}$$

Strain and its types

Strain

$$\text{Strain} = \frac{\text{change in the dimension of the body}}{\text{original dimension of the body}}$$

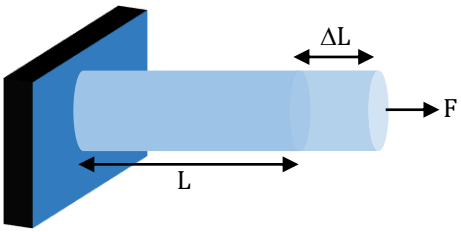
It is a unitless and dimensionless quantity.



(i) Longitudinal strain:

When applied force is ⊥ to cross-section, length changes

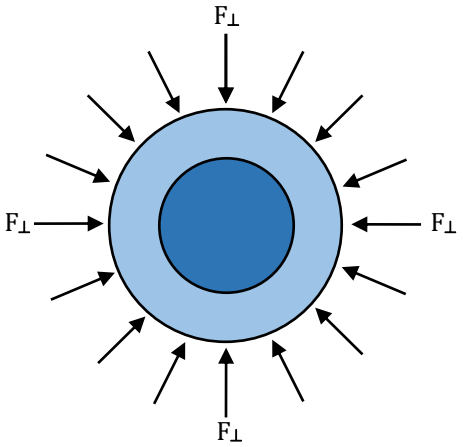
$$\text{Longitudinal strain} = \frac{\text{change in length of the body}}{\text{initial length of the body}} = \frac{\Delta L}{L}$$



(ii) Volumetric strain:

When pressure is applied on body, volume changes

$$\text{Volume strain} = \frac{\text{change in volume of the body}}{\text{original volume of the body}} = \frac{\Delta V}{V}$$



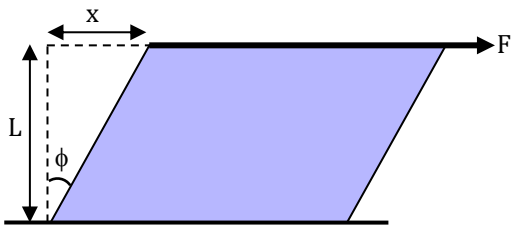
(iii) Shear strain:

When applied force is ∥ to cross-section, shape changes

$\tan\phi = \frac{x}{L}$ (Here ϕ is very small)

$$\phi = \frac{x}{L} = \frac{\text{displacement of upper face relative to the lower face}}{\text{distance between two faces}}$$

ϕ = shear strain OR angle of shear



Relation between angle of twist and angle of shear

When a cylinder of length L and radius r is fixed at one end and a tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ϕ and angle of twist θ.

From diagram

$r\theta = L\phi$

$$\phi = \frac{r\theta}{L}$$

θ = angle of twist

ϕ = angle of shear

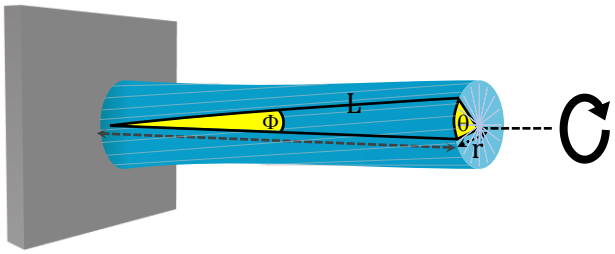


Illustration 4:

A wire of length 1m and radius 2mm is clamped from it's upper end. The lower end is twisted through an angle of 45°. The angle of shear is?

Solution:

$$\phi = \frac{r}{L}\ \theta = \frac{2\times10^{-3}}{1} \times 45° = 0.09°$$

Hooke's law

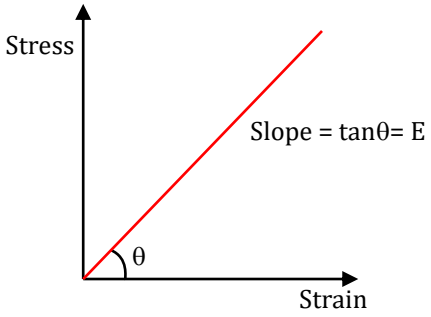
According to this law within the elastic limit the stress produced in a body is directly proportional to the corresponding strain.

Stress α Strain Or **Stress = E × strain**

Here, E = coefficient of elasticity or modulus of elasticity

$$E = \frac{\text{Stress}}{\text{Strain}}$$

The slope of stress & strain graph gives coefficient of elasticity.

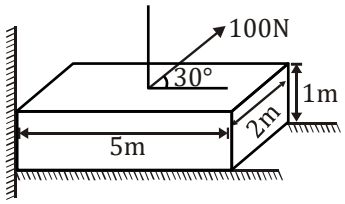


(i) E depends on :
1. Nature of material
2. Impurities
3. Temperature

(ii) E independent from:-
1. Stress
2. Strain

BEGINNER'S BOX-1

1. Find out the longitudinal stress and tangential stress on the fixed block shown in figure.



2. A 2 m long rod of radius 1 cm which is fixed at one end is given a twist of 0.8 radians at the other end. Find the shear strain developed.

3. The maximum stress that can be applied to the material of a wire employed to suspend an elevator is $\frac{3}{\pi} \times 10^8$ N/m². If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 m/s² then calculate the minimum radius of the wire.

4. A human bone is subjected to a compressive force of 5.0×10^5 N. The bone is 25 cm long and has an approximate cross sectional area of 4.0 cm². If the ultimate compressive strength of the bone is 1.70×10^8 N/m², will the bone be compressed or will it break under this force?

5. The breaking stress of steel is 7.9×10^8 N/m² and density is 7.9×10^3 kg/m³. What should be the maximum length of a steel wire so that it may not break under its own weight?

6. A wire can bear a weight of 20 kg before it breaks. If the wire is divided into two equal parts, then each part will support a maximum weight .....

Significance of Modulus of Elasticity

More is the value of Modulus of Elasticity, more is the Elasticity of material.

It means more elastic material will have more tendency to regain its shape under elastic limit deformation (not permanent deformation).

Young's Modulus of Elasticity

Within elastic limit, the ratio of longitudinal stress to longitudinal strain is called Young's modulus of elasticity.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L}$$

Unit of Y: N/m^2 Dimensions of Y: $[M^1L^{-1}T^{-2}]$

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L} \Rightarrow F = \frac{YA}{L}\Delta L$$

Note : For spring : F = kx ...(i)

For wire : $F = \frac{YA}{L}\Delta L$ ...(ii)

From (i) and (ii), $k = \frac{YA}{L}$

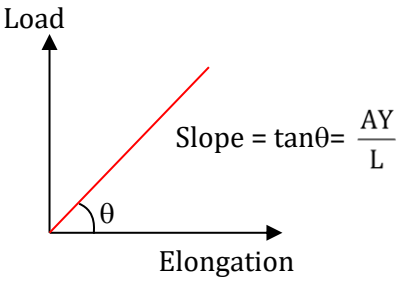


Illustration 5:

A copper wire of negligible mass, length 1 m and cross-sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of mass 1kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad/s. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus of copper.

Solution:

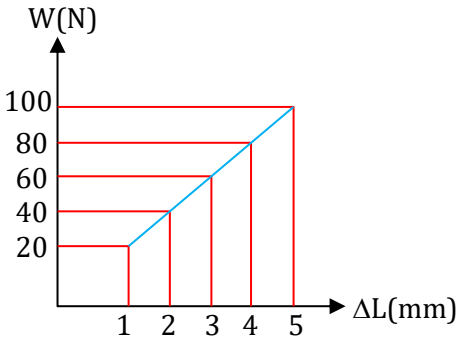
Centripetal force $F = m\omega^2 r$

$\therefore$ Stress in the wire = $\frac{F}{A} = \frac{m\omega^2 r}{A}$ and strain in the wire = $\frac{\Delta L}{L}$

Young's modulus $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{m\omega^2 L \cdot L}{A.\Delta L} = \frac{1\times(20)^2\times(1)^2}{10^{-6}\times10^{-3}} = 4 \times 10^{11}$ N/m^2

Illustration 6:

The graph shows the extension of a wire of length 1m suspended from a roof at one end and with a load W connected to the other end. If the cross sectional area of the wire is 1 mm^2, then the Young's modulus of the material of the wire is



Solution:

$$Y = \frac{F/A}{\Delta L/L} = \frac{WL}{A\Delta L} \Rightarrow \frac{W}{\Delta L} = \frac{YA}{L} = \text{slope}$$

$$\Rightarrow Y = \frac{L}{A} \times (\text{slope}) = \frac{1}{10^{-6}}\left(\frac{40-20}{(2-1)\times10^{-3}}\right) = 2 \times 10^{10}\ N/m^2$$

Illustration 7:

Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both the wires, find the ratio of increase in their lengths.

Solution

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \Rightarrow \Delta L = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y} = \frac{\Delta L_1}{\Delta L_2} = \frac{4FL_1}{\pi d_1^2 Y} \times \frac{\pi d_2^2 Y}{4FL_2} = \frac{L_1}{L_2} \times \frac{d_2^2}{d_1^2} = \frac{1}{2} \times \frac{1}{(2)^2} = \frac{1}{8}$$

Illustration 8.

Calculate the force required to increase the length of a steel wire of cross-sectional area $10^{-6}m^2$ by 0.5%. given: Y(for steel) = 2×10^{11} N/m^2.

Solution:

$$\frac{\Delta L}{L} \times 100 = 0.5\% \Rightarrow \frac{\Delta L}{L} = 5 \times 10^{-3}$$

So, $F = YA\frac{\Delta L}{L} = 2 \times 10^{11} \times 10^{-6} \times 5 \times 10^{-3} = 10^3$ N

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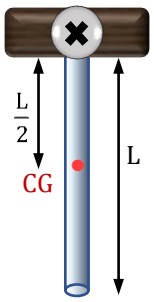
Elongation in hanging wire due to its own weight

Since the weight can assumed to be act at the centre of gravity, therefore



$\therefore$ The original length will be taken as $\frac{L}{2}$ $\therefore$ $Y = \frac{Mg \times \frac{L}{2}}{A \times \Delta L} \Rightarrow \Delta L = \frac{MgL}{2AY}$

But, M = (LA) ρ $\therefore$ $\Delta L = \frac{LA\rho gL}{2AY}$ or $\Delta L = \frac{\rho g L^2}{2Y}$

Illustration 9:

A rubber rope of length 8 m is hung from the ceiling of a room. What is the increase in length of the rope due to its own weight? (Given : Young's modulus of elasticity of rubber = 5×10^6 N m^{-2} and density of rubber = 1.5×10^3 kg/m^3 and g = 10 m/s^2).

Solution:

$\Delta L = \frac{\rho g \ell^2}{2Y} = \frac{1.5 \times 10^3 \times 10 \times 8 \times 8}{2 \times 5 \times 10^6} = 9.6 \times 10^{-2}$ m = $9.6 \times 10^{-2} \times 10^3$ mm = 96 mm.

Bulk modulus of elasticity 'K' or 'B'

Within elastic limit, the ratio of the volumetric stress (i.e., change in pressure) to the volumetric strain is called Bulk modulus of elasticity.

$K \text{ or } B = \frac{\text{volumetric stress}}{\text{volumetric strain}} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$

The negative sign indicates a decrease in volume with an increase in pressure and vice-versa.

Unit of K: N/m^2 or Pascal

Compressibility 'C'

The reciprocal of Bulk modulus of elasticity is defined as compressibility.

$C = \frac{1}{K}$ SI unit of C: m^2/N or $pascal^{-1}$

Illustration 10:

A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the Bulk modulus of the material of the sphere. Given: density of sea water is 1 g/cm^3, g = 980 cm/s^2.

Solution:

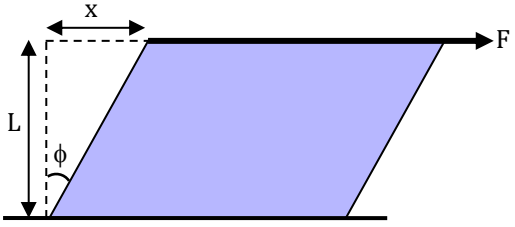
$\frac{|\Delta V|}{V} = \frac{0.01}{100}$, h = 1 km = 10^5 cm, ρ = 1 g/cm^3 ; $\Delta P = h\rho g = 10^5 \times 1 \times 980$ dyne/cm^2, K = ?

$K = \frac{\Delta P}{|\Delta V|/V} = \frac{\Delta P \times V}{|\Delta V|} = \frac{10^5 \times 980 \times 100}{0.01}$ dyne/cm^2 = 9.8×10^{11} dyne/cm^2.

Modulus of Rigidity 'η'

Within elastic limit, the ratio of shearing stress to shearing strain is called modulus of rigidity of a material.

$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \left(\frac{F_{tangential}}{\frac{A}{\phi}} \right) = \frac{F_{tangential}}{A\phi}$



Note: Angle of shear 'Φ' is always taken in radians

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Material / E	Solid	Liquid	Gas	Ideal rigid body
Young's modulus	defined	Not defined (0)	Not defined (0)	Infinite
Bulk modulus	defined	defined	defined	infinite
Shear modulus	defined	Not defined (0)	Not defined (0)	infinite

Work done in stretching a wire (Potential energy of a stretched wire)

When a wire of length 'L' is stretched by an external force F, then work has to be done against the restoring force this work is stored as potential energy of wire.

For spring :W.D.= Elastic Potential energy = $\frac{1}{2}kx^2$

For wire :W = PE = $\frac{1}{2}\left(\frac{YA}{L}\right)(\Delta L)^2$

$$EPE = \frac{1}{2}\left(\frac{YA}{L}\Delta L\right)(\Delta L) = \frac{1}{2}F(\Delta L)$$

$$EPE = \frac{1}{2}\frac{F}{A}\left(\frac{\Delta L}{L}\right)(AL)$$

$$EPE = \frac{1}{2}(\text{stress})\ (\text{strain})\ (\text{volume})$$

$$EPE = \frac{1}{2}Y(\text{strain})^2(\text{volume})$$

$$EPE = \frac{1}{2}\left(\frac{(\text{stress})^2}{Y}\right)(\text{volume})$$

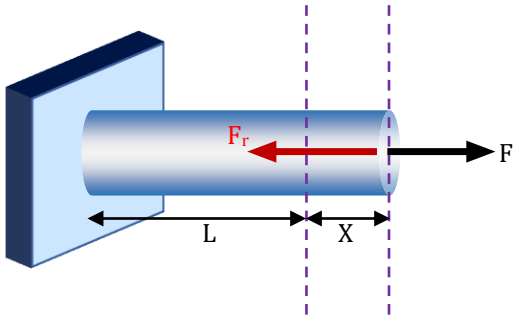

Illustration 11:

A rubber cord has a cross–sectional area 1 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a mass of 5 g. Young's modulus for rubber is 5×10^8 N/m^2. Find the tension in the cord and velocity of the mass.

Solution:

Tension in the cord = $\frac{YA}{L}\Delta L = \frac{5\times10^8\times1\times10^{-6}\times2\times10^{-2}}{10\times10^{-2}}$ = 100 N.

When the mass is released, elastic energy stored = Kinetic energy of mass

$$\frac{1}{2}F\times\Delta L = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{F\times\Delta L}{m}} = \sqrt{\frac{100\times2\times10^{-2}}{5\times10^{-3}}} = 20 \text{ m/s.}$$

Illustration 12:

Young modulus of elasticity of brass is 10^{11} N/m^2. The increase in its energy on pressing a rod of length 0.1 m and cross–sectional area 1 cm^2 made of brass with a force of 10 kg along its length, will be .....

Solution:

Increase in energy = $\frac{1}{2}F\times\Delta L = \frac{1}{2}\times F\times\frac{FL}{AY} = \frac{1}{2}\times(10\times10)^2\times\frac{0.1}{10^{-4}\times10^{11}} = 5\times10^{-5}$ J

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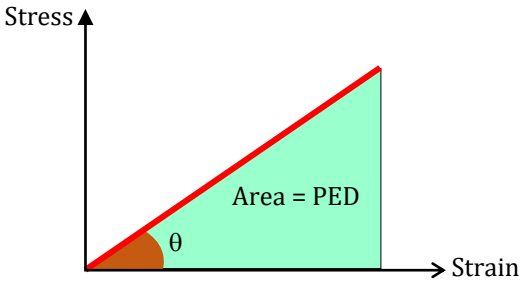
Potential energy density

Potential energy per unit volume stored in the wire

$$PED = \frac{PE}{vol}$$

SI unit : J/m^3 or N/m^2

$$PED = \frac{1}{2}(stress)(strain) = \frac{1}{2}Y(strain)^2 = \frac{1}{2}\left(\frac{(stress)^2}{Y}\right)$$



Note: Slope = tanθ = E = coefficient of elasticity

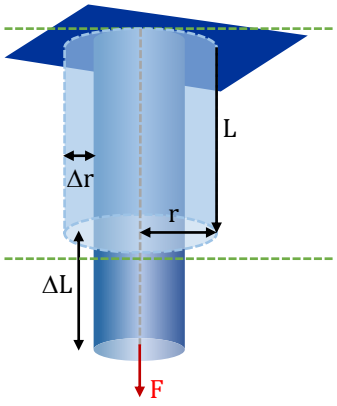
$$Area = \frac{1}{2} \times stress \times strain = Potential\ Energy\ Density$$

Poisson's Ratio (σ)

$$longitudinal\ strain = \frac{\Delta L}{L};\ lateral\ strain = \frac{\Delta r}{r}$$

Within elastic limit, the ratio of lateral strain to the longitudinal strain is called poisson's ratio.

$$\sigma = \frac{lateral\ strain}{longitudinal\ strain} \quad \Rightarrow \quad \sigma = -\frac{\Delta r / r}{\Delta L / L}$$



$-1 \le \sigma \le 0.5$ (theoretical limit)

$\sigma \approx 0.2 - 0.4$ (experimental limit)

Interatomic Force Constant

Interatomic force constant k is equal to the product of Young's modulus of the material of the wire and the normal distance r_0 between the atoms of the wire.

k or $k_a = Y.\ r_0$

Y = Young's modulus; r_0 = interatomic distance

Illustration 13:

Young modulus of elasticity of steel is 2×10^{11} N/m^2. If interatomic distance for steel is 3.2 Å, then find the interatomic force constant.

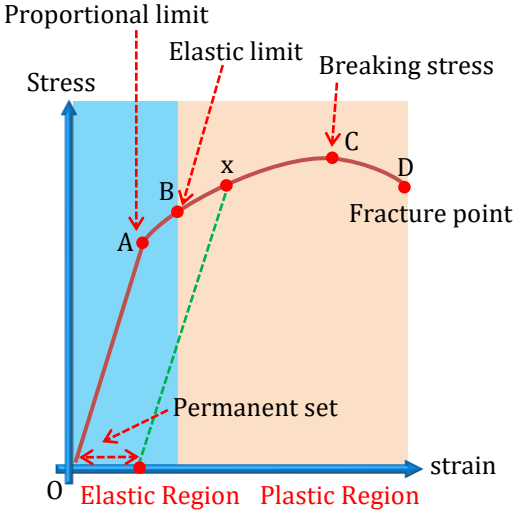
Solution:

$k = Y \times r_0 = 2 \times 10^{11} \times 3.2 \times 10^{-10} = 64$ N/m.

Relation between Y,K,η and σ : (To be remembered)

1. $Y = 3K(1 - 2\sigma)$ 2. $Y = 2\eta(1 + \sigma)$ 3. $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$

Stress Strain relationship and Graphical analysis



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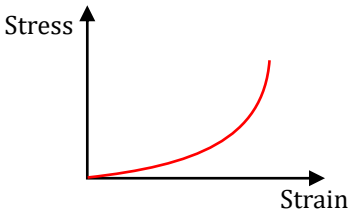
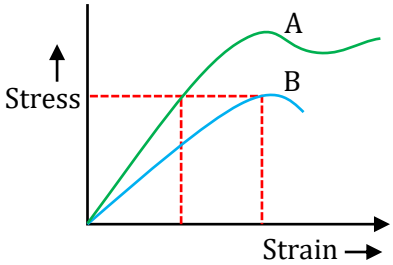
1. In the region between O to A the curve is linear. Hooke's law is obeyed. In this region the solid behaves as an elastic body.
2. In the region A to B stress and strain are not proportional but body regains its original shape and size when the load is removed. Point B is known as the elastic limit.
3. If the load is increased further the strain developed increases rapidly than stress.
4. In the region from B to D, when load is removed the body does not regain its original dimensions. Even when stress is zero the strain is not zero. The material is said to have a permanent set. The region beyond point B is known as the plastic region.
5. Point C corresponds to the ultimate tensile strength; beyond this point additional strain is produced by even a reduced applied force and fracture occurs at point D.
6. If plastic region is large then material will be ductile.
7. If plastic region is small then material will be brittle.



Note: For some materials elastic region is very large and the material does not obey Hooke's law over most of the region. These are called elastomers e.g. Tissue of Aorta, rubber, etc.

Illustration 14:

The stress versus strain graphs for two materials A and B are shown below. Explain the following



(a) Which material has greater Young's modulus?
(b) Which material is more ductile?
(c) Which material is more brittle?
(d) Which of the two is stronger material?

Solution:

(a) Material A has greater value of Young's modulus, because slope of A is greater than that of B.
(b) Material A is more ductile because there is a large plastic deformation range between the elastic limit and the breaking point.
(c) Material B is more brittle because the plastic region between the elastic limit and breaking point is small.
(d) Strength of a material is determined by the stress required to cause fracture. Material A is stronger than material B.

BEGINNER'S BOX-2

1. A stress of 20×10^8 N/m² is developed when the length of a wire is doubled. Its Young's modulus of elasticity in N/m² will be .....
2. The ratio of lengths of two wires made up of the same material is 3 : 1 and the ratio of their radii is 1 : 3. The ratio of increments of lengths on account of suspending the same weight will be .....
3. The following four wires are made of same material. Which one will have the largest elongation when subjected to the same tension?
   (1) Length 500 cm, diameter 0.05 mm.
   (2) Length 200 cm, diameter 0.02 mm.
   (3) Length 300 cm, diameter 0.03 mm.
   (4) Length 400 cm, diameter 0.01 mm.

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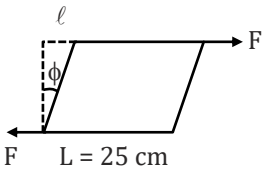
4. The bulk's modulus of copper is 138×10^9 Pa. The additional pressure generated in an explosion chamber is 345×10^6 Pa. Then the percentage change in the volume of a piece of copper placed in this chamber will be .....

5. The compressibility of water per unit atmospheric pressure is 4×10^{-5}. Decrease in the volume of 100 cm^3 water of volume at 100 atmospheric pressure will be .....

6. Two parallel and opposite forces, each of magnitude 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block of side 25 cm. If the shear modulus for the metal is 8×10^{10} Pa, then the displacement of the upper surface relative to the lower surface will be .....



7. Young modulus of elasticity of brass is 10^{11} N/m^2. The increase in its energy on pressing a rod of length 0.1 m and cross–sectional area 1 cm^2 made of brass with a force of 10 kg along its length, will be .....

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